Lecture 12

AO Control Theory

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with <u>many</u> thanks to Don Gavel and Don Wiberg
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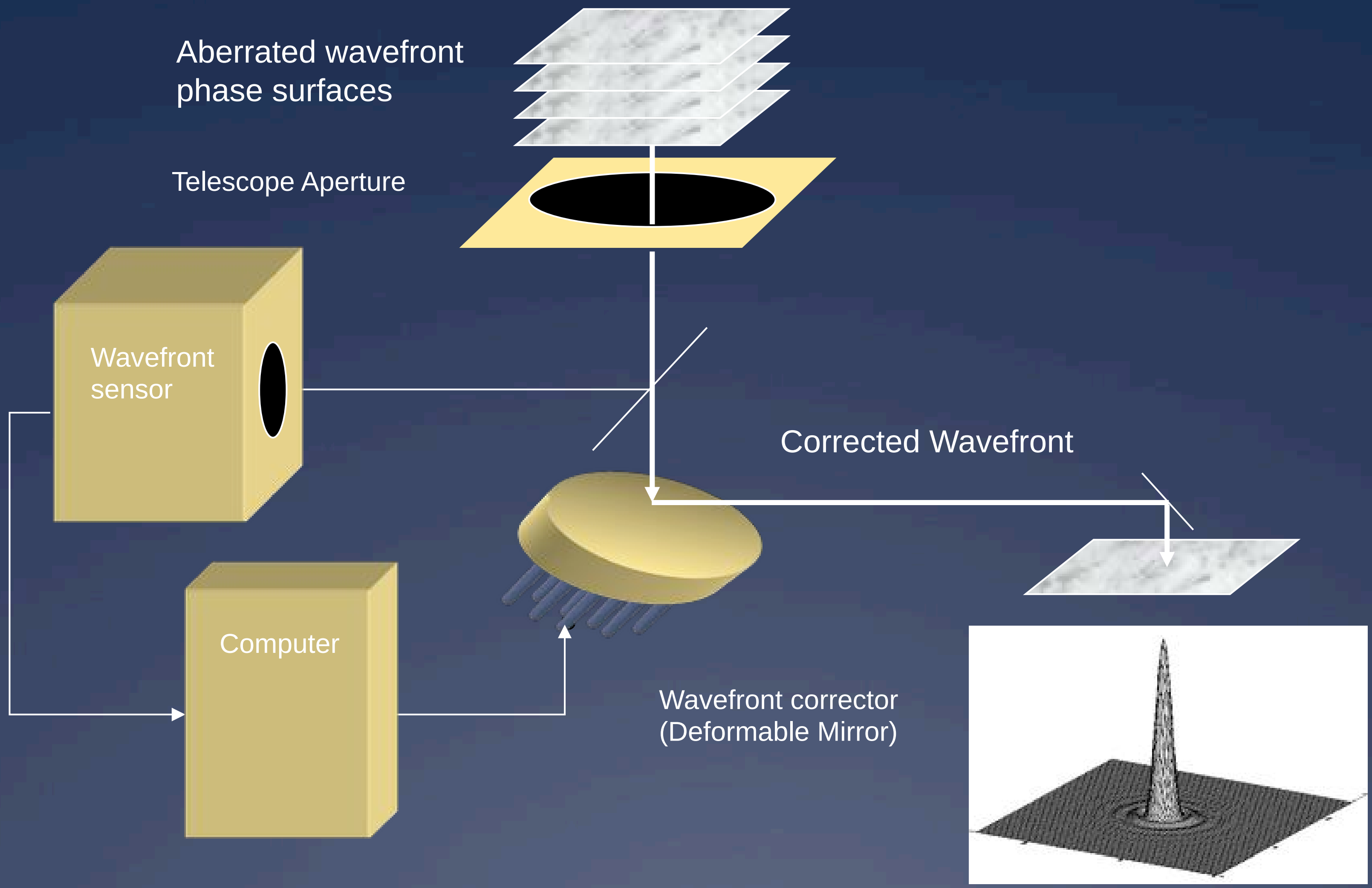
What are control systems?

- Control is the process of making a system variable adhere to a particular value, called the reference value.
- A system designed to follow a changing reference is called tracking control or servo.

Outline of topics

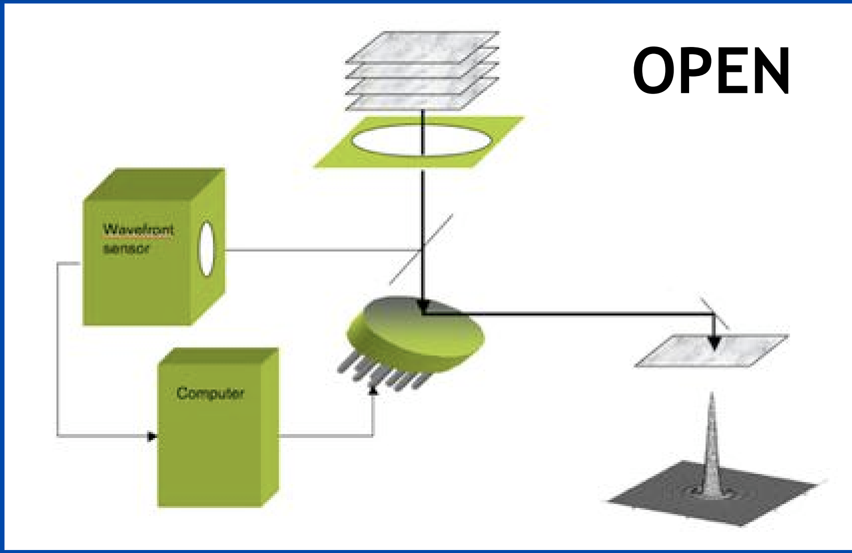
- What is control?
  - The concept of closed loop feedback control
- A basic tool: the Laplace transform
  - Using the Laplace transform to characterize the time and frequency domain behavior of a system
  - Manipulating *Transfer functions* to analyze systems
- How to predict performance of the controller

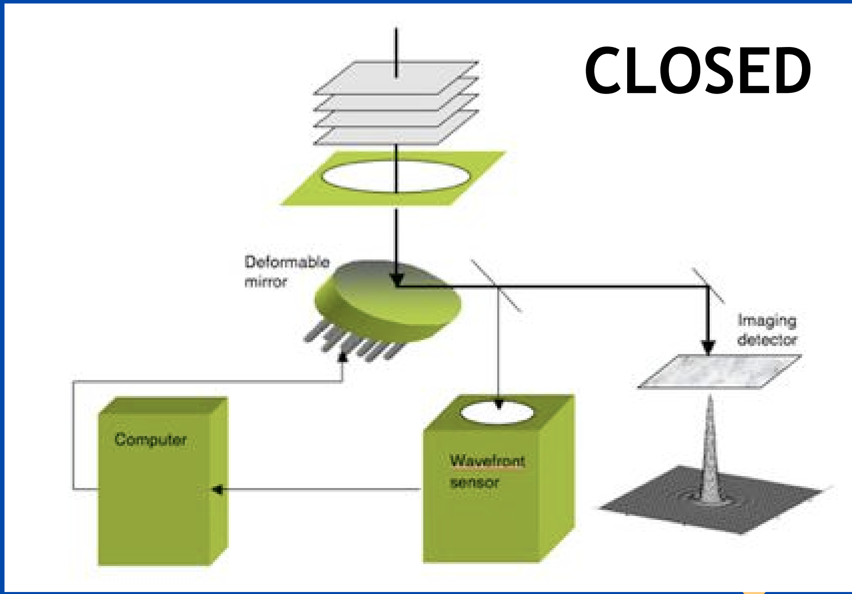
Adaptive Optics Control



Differences between open-loop and closed-loop control systems

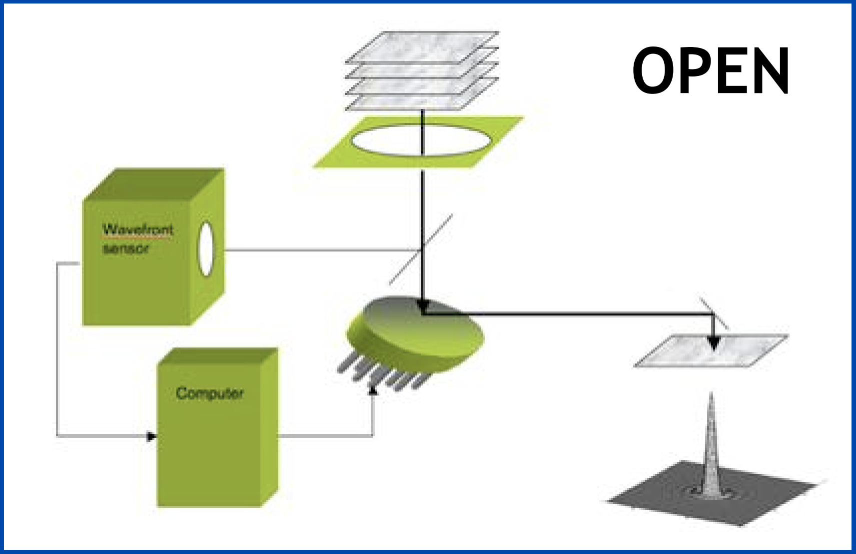
- Open-loop: control system uses no knowledge of the output
- Closed-loop: the control action is dependent on the output in some way
- "Feedback" is what distinguishes open from closed loop
- What other examples can you think of?





More about open-loop systems

- Need to be carefully calibrated ahead of time:
- Example: for a deformable mirror, need to know exactly what shape the mirror will have if the n actuators are each driven with a voltage V_n
- Question: how might you go about this calibration?



Some Characteristics of Closed- Loop Feedback Control

- Increased accuracy (gets to the desired final position more accurately because small errors will get corrected on subsequent measurement cycles)
- Less sensitivity to nonlinearities (e.g. hysteresis in the deformable mirror) because the system is always making small corrections to get to the right place
- Reduced sensitivity to noise in the input signal
- BUT: can be unstable under some circumstances (e.g. if gain is too high)

Historical control systems: float valve

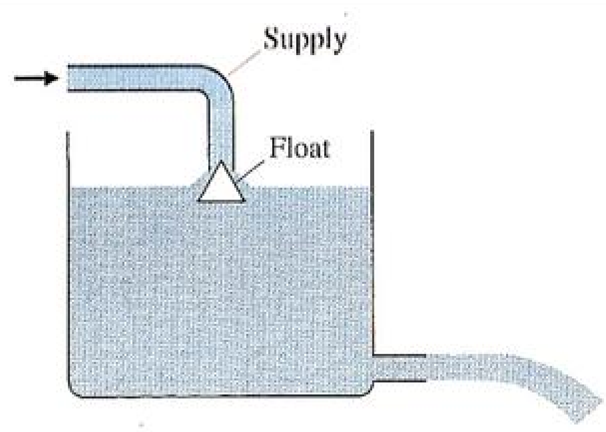


Figure 1.7
Early historical control of liquid level and flow

Credit: Franklin, Powell, Emami-Naeini

- As liquid level falls, so does float, allowing more liquid to flow into tank
- As liquid level rises, flow is reduced and, if needed, cut off entirely
- Sensor and actuator are both "contained" in the combination of the float and supply tube

Block Diagrams: Show Cause and Effect

Fig. 2-2

Credit: DiStefano et al. 1990

- **Pictorial representation of cause and effect**
- **Interior of block shows how the input and output are related.**
- **Example b: output is the time derivative of the input**

"Summing" Block Diagrams are circles

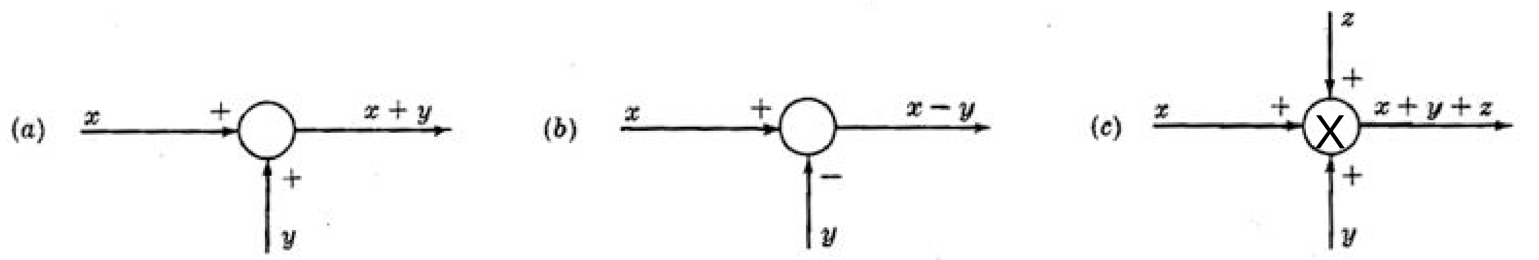


Fig. 2-3

Credit: DiStefano et al. 1990

- **Block becomes a circle or "summing point"**
- **Plus and minus signs indicate addition or subtraction (note that "sum" can include subtraction)**
- **Arrows show inputs and outputs as before**
- **Sometimes there is a cross in the circle**

A home thermostat from a control theory point of view

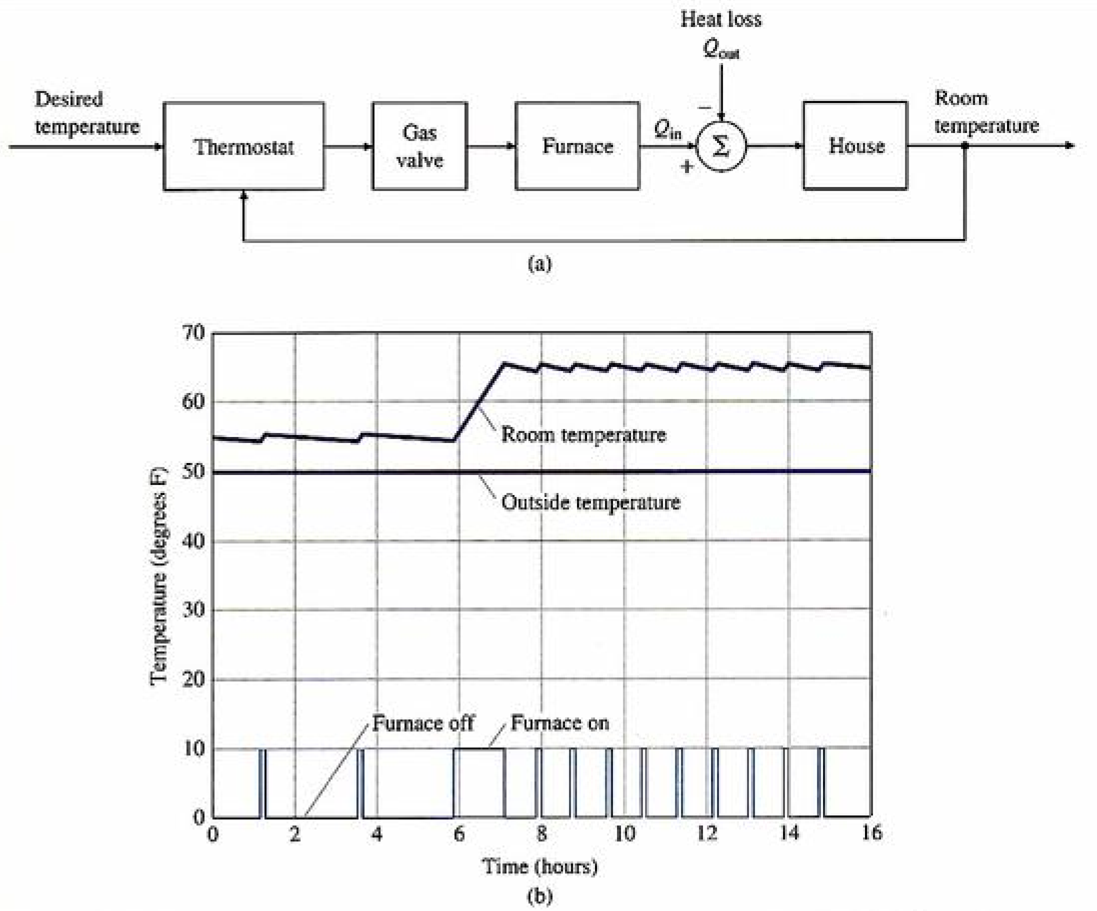
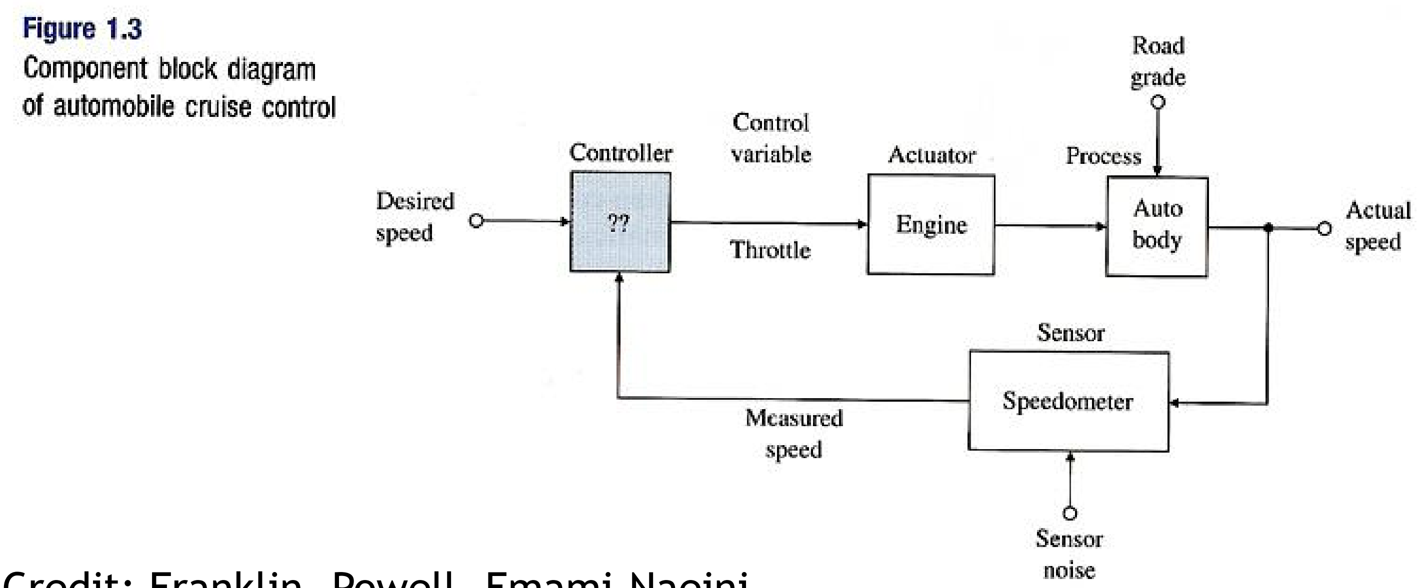


Figure 1.1 (a) Component block diagram of a room temperature control system (b) Plot of room temperature and furnace action

Credit: Franklin, Powell, Emami-Naeini

Block diagram for an automobile cruise control



Credit: Franklin, Powell, Emami-Naeini

Example 1

- Draw a block diagram for the equation

$$x_3 = \frac{d^2 x_2}{dt^2} + \frac{dx_1}{dt} - x_1$$

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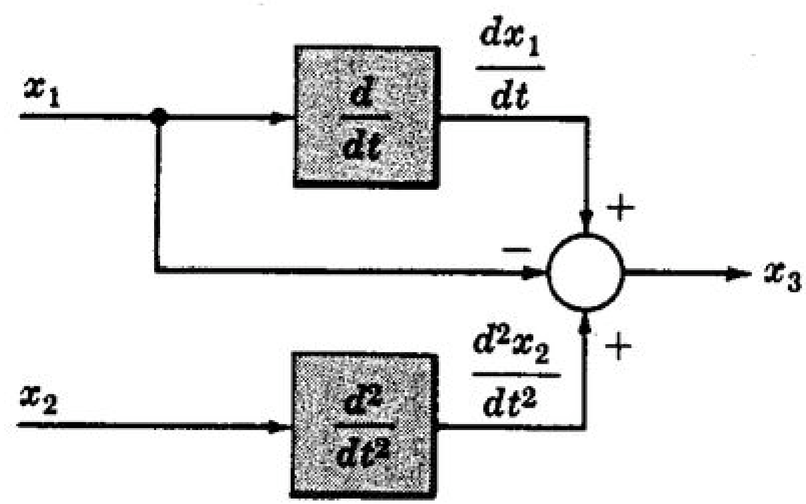

Credit: DiStefano et al. 1990

Example 2

- Draw a block diagram for how your eyes and brain help regulate the direction in which you are walking

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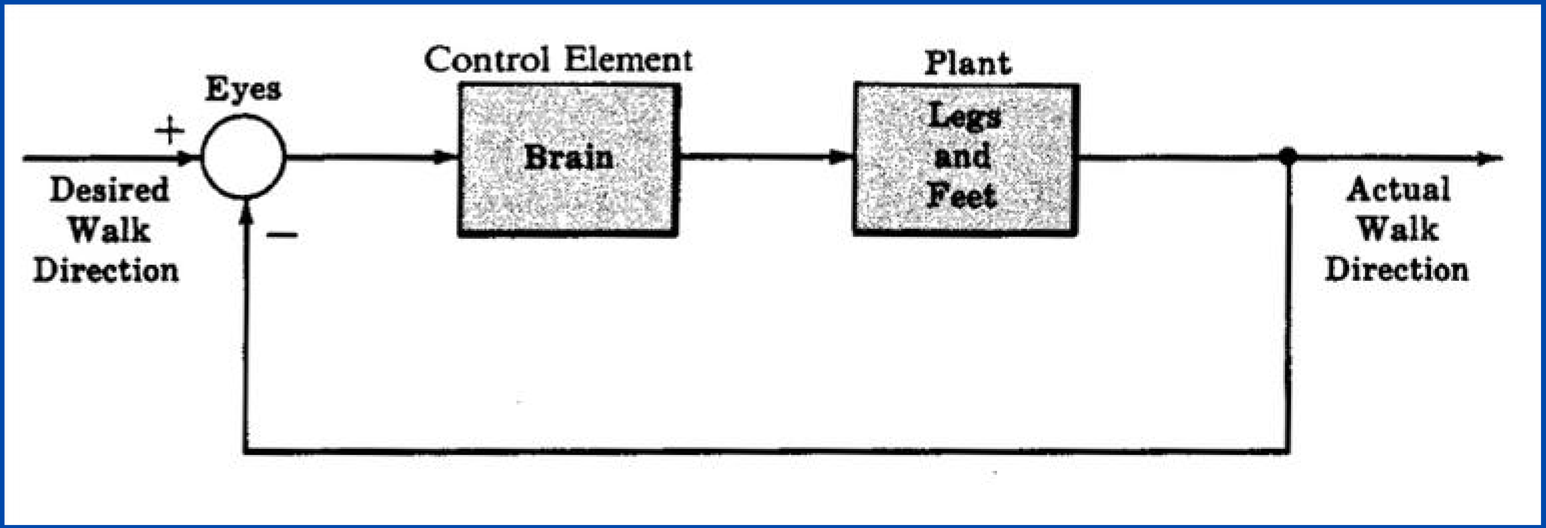


Credit: DiStefano et al. 1990

Summary so far

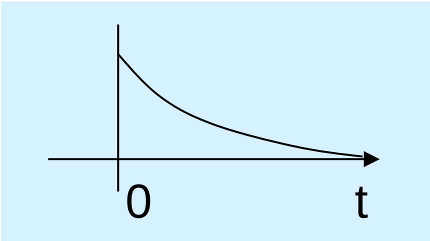
- Distinction between open loop and closed loop
  - Advantages and disadvantages of each
- Block diagrams for control systems
  - Inputs, outputs, operations
  - Closed loop vs. open loop block diagrams

The Laplace Transform Pair

$$H(s) = \int_0^{\infty} h(t) e^{-st} dt$$

$$h(t) = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} H(s) e^{st} ds$$

- <u>Example</u>: decaying exponential

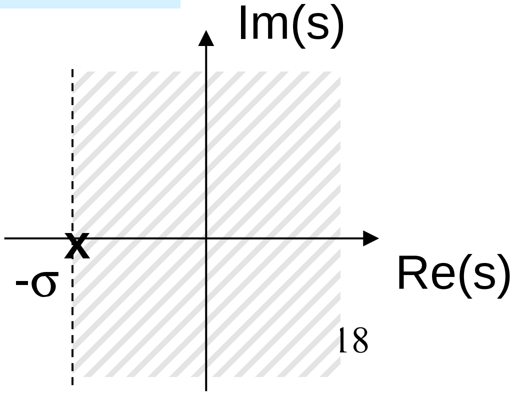


Transform: $h(t) = e^{-\sigma t}$

$$H(s) = \int_0^{\infty} e^{-(s+\sigma)t} dt$$

$$= \frac{-1}{s+\sigma} e^{-(s+\sigma)t} \Big|_0^{\infty}$$

$$= \frac{1}{s+\sigma}; \qquad \mathrm{Re}(s) > -\sigma$$



The Laplace Transform Pair

<u>Example</u> (continued), decaying exponential

Inverse Transform:

$$h(t) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} e^{st}\frac{1}{s+\sigma}\,ds$$

$$= \frac{1}{2\pi i}\int_{-\pi}^{\pi} \rho^{-1}e^{-\sigma t}\; i\rho\, d\theta$$

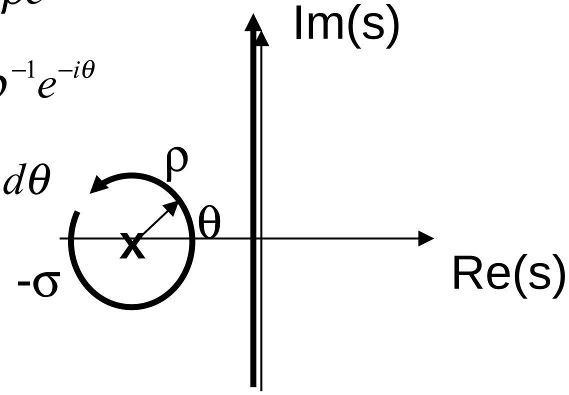
$$= e^{-\sigma t}\frac{1}{2\pi i}\int_{-\pi}^{\pi} i\, d\theta$$

$$= e^{-\sigma t}$$

$$s = -\sigma + \rho e^{i\theta}$$

$$\frac{1}{s+\sigma} = \rho^{-1}e^{-i\theta}$$

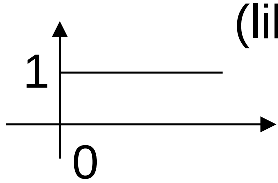
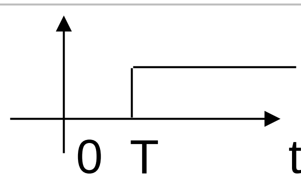
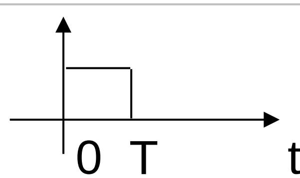
$$ds = i\rho e^{i\theta}d\theta$$



The above integration makes use of the <u>Cauchy Principal Value Theorem</u>:

If F(s) is *analytic* then $\oint F(s)\frac{1}{s-a}ds = 2\pi i F(a)$

Laplace Transform Pairs

	h(t)	H(s)
unit step	 (like $\lim_{s \to 0} e^{-st}$)	$\frac{1}{s}$
	$e^{-\sigma t}$	$\frac{1}{s+\sigma}$
	$e^{-\sigma t}\cos(\omega t)$	$\frac{1}{2}\left(\frac{1}{s+\sigma-i\omega}+\frac{1}{s+\sigma+i\omega}\right)$
	$e^{-\sigma t}\sin(\omega t)$	$\frac{1}{2i}\left(\frac{1}{s+\sigma-i\omega}-\frac{1}{s+\sigma+i\omega}\right)$
delayed step		$\frac{e^{-sT}}{s}$
unit pulse		$\frac{1-e^{-sT}}{s}$

Laplace Transform Properties (1)

$$L\{\ \alpha h(t)+\beta g(t)\ \}=\alpha H(s)+\beta G(s)$$

Linearity

$$L\{\ h(t+T)\ \}=e^{sT}H(s)$$

Time-shift $(T\leq 0)$

$$L\{\ \delta(t)\ \}=1$$

Dirac delta function transform (“sifting” property)

$$L\left\{\int_0^t h(t')g(t-t')dt'\right\}=H(s)\,G(s)$$

Convolution

$$L\left\{\int_0^t h(t')\,\delta(t-t')dt'\right\}=H(s)$$

Impulse response

$$\int_0^t h(t-t')\,e^{i\omega t'}dt'=H(i\omega)\,e^{i\omega t}$$

Frequency response

Laplace Transform Properties (2)

System Block Diagrams

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) \qquad\qquad X(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s)$$

convolution of input *x(t)* with impulse response *h(t)* $\longleftrightarrow$

- Product of input spectrum *X(s)* with frequency response *H(s)*
- *H(s)* in this role is called the *transfer function*

$$L\left\{ \int_0^t h(t')x(t-t')dt' \right\} = H(s)X(s) = Y(s)$$

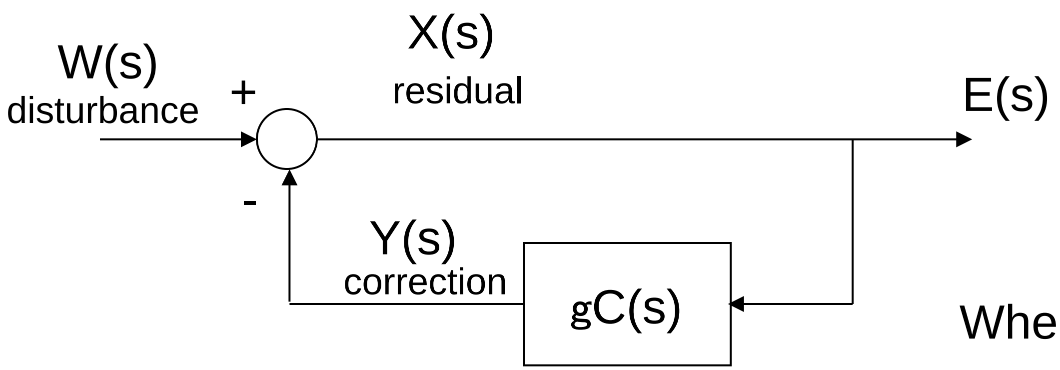
Conservation of Power or Energy

$$\int_0^\infty [h(t)]^2\, dt = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \left|H(s)\right|^2 ds$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|H(i\omega)\right|^2 d\omega$$

“Parseval’ s Theorem”

Closed loop control (simple example, H(s)=1)



Where $g \equiv$ gain

Our goal will be to suppress X(s) (residual) by high-gain feedback so that Y(s)~W(s)

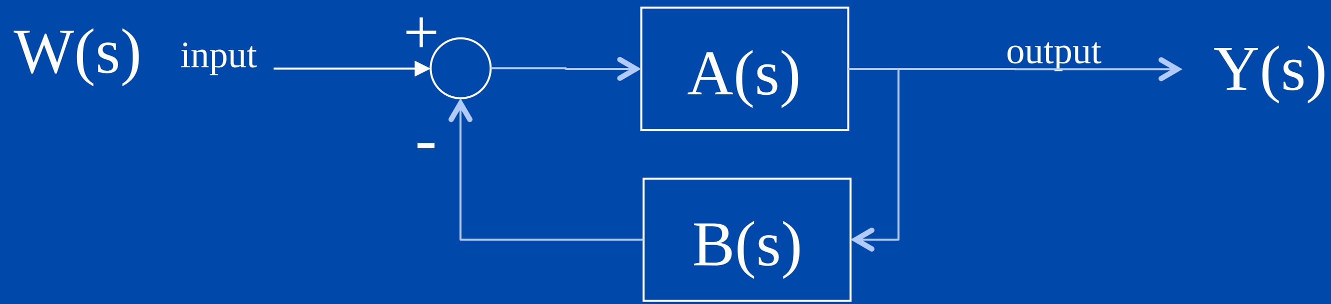
$$E(s) = W(s) - gC(s)E(s)$$

solving for E(s),

$$E(s) = \frac{W(s)}{1 + gC(s)}$$

Note: for consistency "around the loop," the units of the gain g must be the inverse of the units of C(s).

Back Up: Control Loop Arithmetic



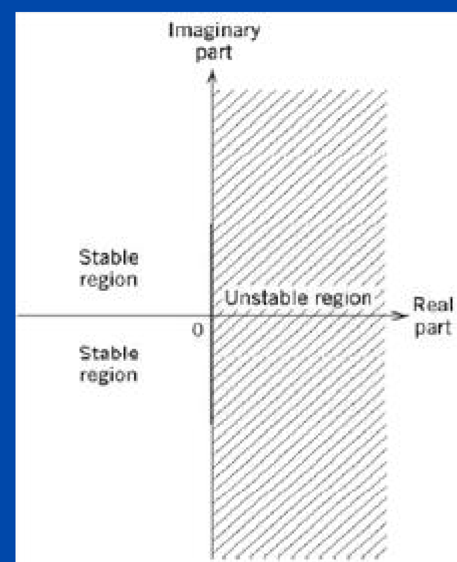
$$Y(s) = A(s)W(s) - A(s)B(s)Y(s)$$

$$Y(s) = \frac{A(s)W(s)}{1 + A(s)B(s)}$$

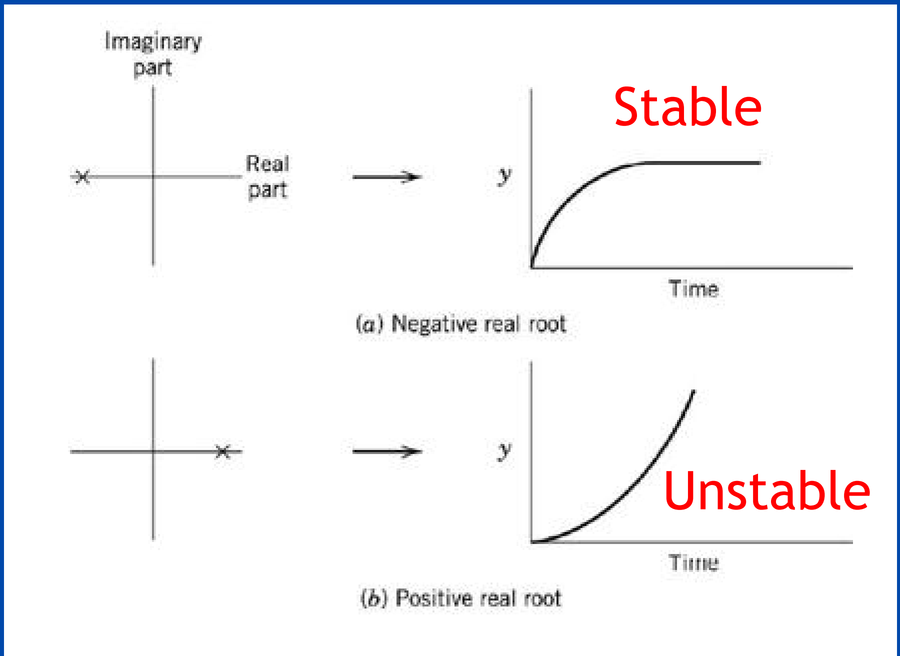
Unstable if any roots of

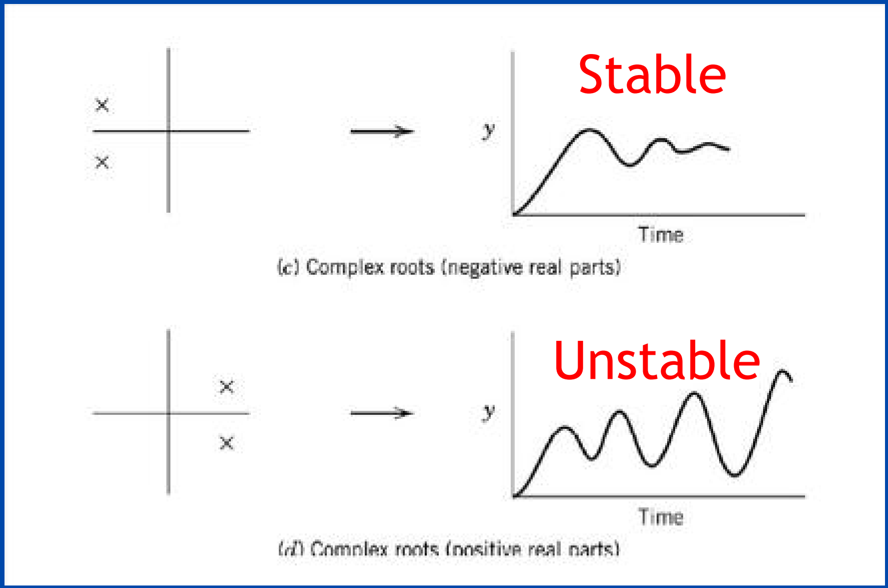
$1+A(s)B(s) = 0$

are in right-half of the s-plane: exponential growth $\exp(st)$

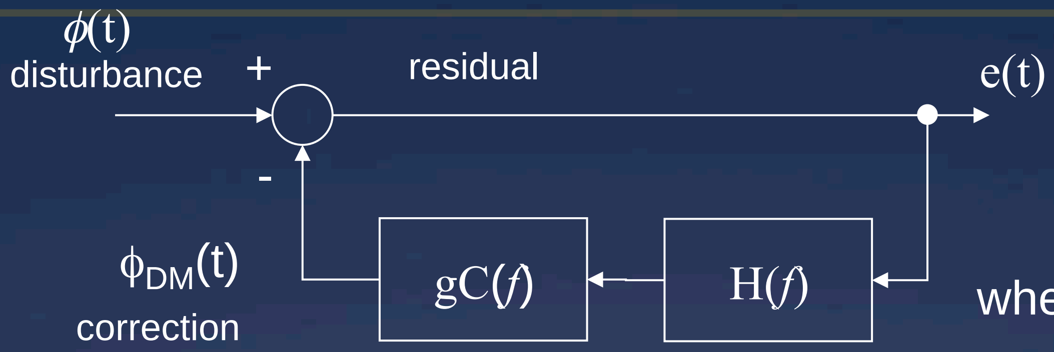


Stable and unstable behavior





Block Diagram for Closed Loop Control



where g = loop gain

H(*f*) = Camera Exposure x DM Response x Computer Delay
C(*f*) = Controller Transfer Function

Our goal will be to find a C(*f*) that suppress e(t) (residual) so that ϕ_{DM} tracks ϕ

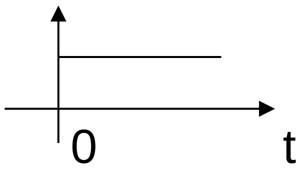
$$e(f) = \frac{1}{1 + gC(f)H(f)}\,\phi(f)$$

We can design a filter, C(*f*), into the feedback loop to:

a) Stabilize the feedback (i.e. keep it from oscillating)
b) Optimize performance

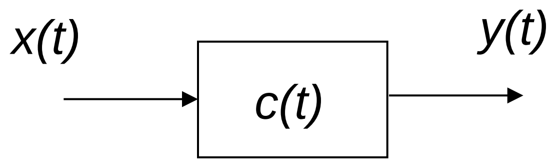
The *integrator*, one choice for C(s)

A *system* whose impulse response is the unit step

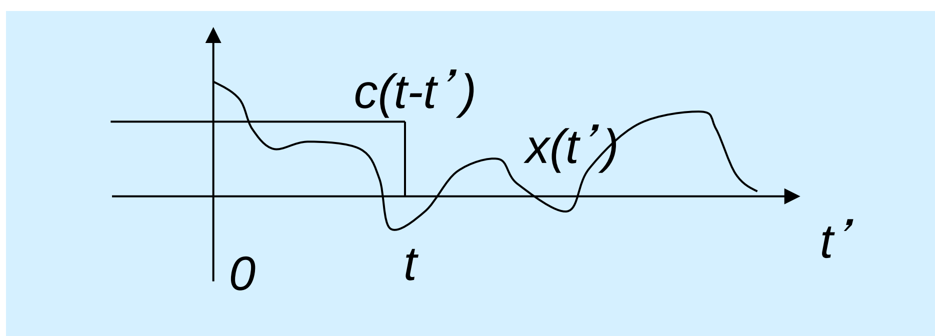


$$c(t)=\begin{cases}0 & t<0\\ 1 & t\geq 0\end{cases} \quad \leftrightarrow \quad C(s)=\frac{1}{s}$$

acts as an integrator to the input signal:



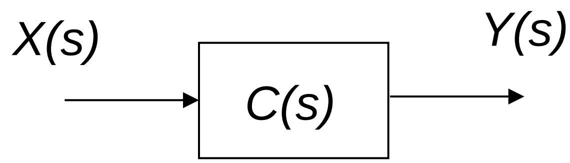
$$y(t)=\int_0^t c(t-t')x(t')dt' = \int_0^t x(t')dt'$$



that is, *C(s)* integrates the past history of inputs, *x(t)*

The Integrator, continued

In Laplace terminology:

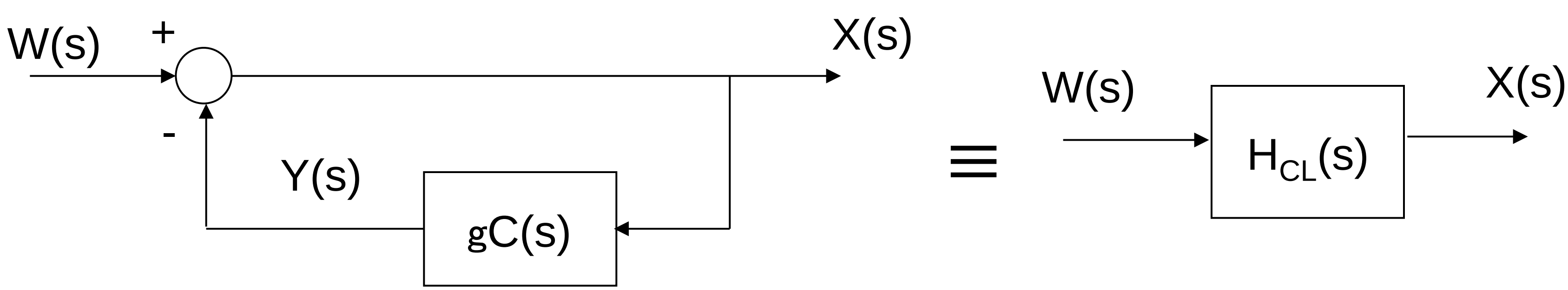


$$Y(s) = \frac{X(s)}{s}$$

An integrator has high gain at low frequencies, low gain at high frequencies.

Write the input/output transfer function for an integrator in closed loop:

The closed loop transfer function with the integrator in the feedback loop is:



$$C(s) = \frac{1}{s} \quad \Rightarrow \quad X(s) = \frac{W(s)}{1+g/s} = \left(\frac{s}{s+g}\right) W(s) = H_{CL}(s) W(s)$$

closed loop transfer function

output (e.g. residual wavefront to science camera)

input disturbance (e.g. atmospheric wavefront)

The integrator in closed loop (1)

$$H_{CL}(s) = \frac{s}{s+g}$$

$H_{CL}(s)$, viewed as a sinusoidal response filter:

$$H_{CL}(s) \rightarrow 0 \quad \text{as} \quad s \rightarrow 0$$

DC response = 0
("Type-0" behavior)

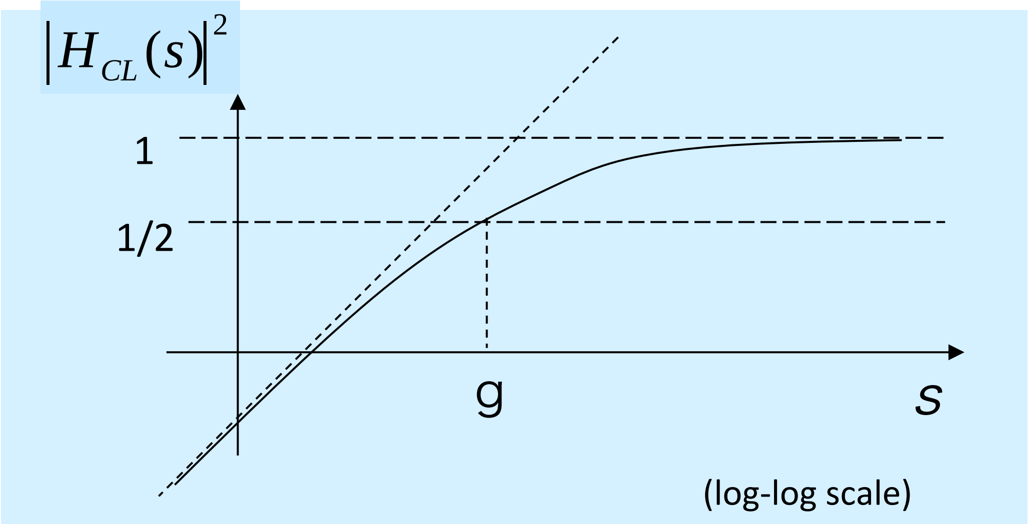
$$H_{CL}(s) \rightarrow 1 \quad \text{as} \quad s \rightarrow \infty$$

High-pass behavior

and the "break" frequency (transition from low freq to high freq behavior) is around $s \sim g$

The integrator in closed loop (2)

The break frequency is often called the "half-power" frequency



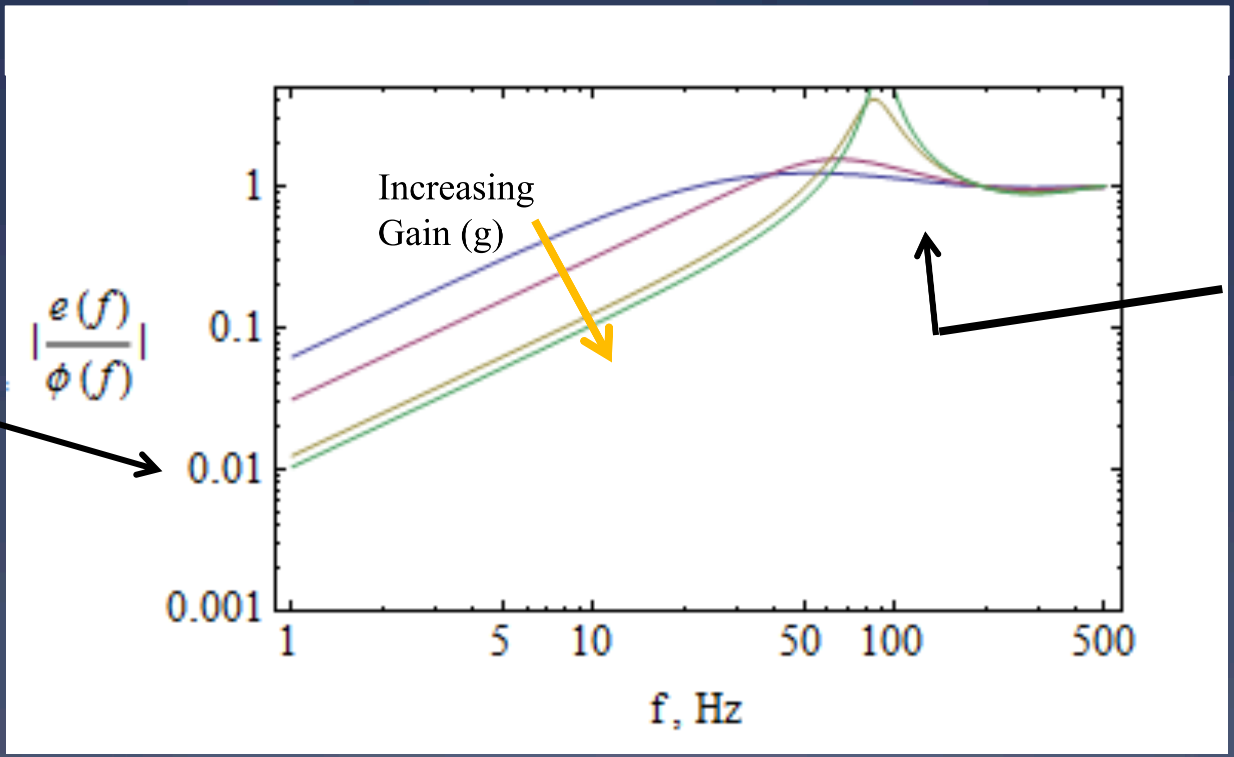
$$H_{CL}(s) = \frac{s}{s+g}$$

- Note that the gain, g, is the *bandwidth* of the controller:
- Frequencies below g are rejected, frequencies above g are passed.
- By convention, g is known as the gain-bandwidth product.

Disturbance Rejection Curve for Feedback Control With Compensation

$$\frac{e(f)}{\phi(f)} = \frac{1}{1 + gC(f)H(f)}$$

Much better rejection



Starting to resonate

Assume that residual wavefront error is introduced by only two sources

1. Failure to completely cancel atmospheric phase distortion

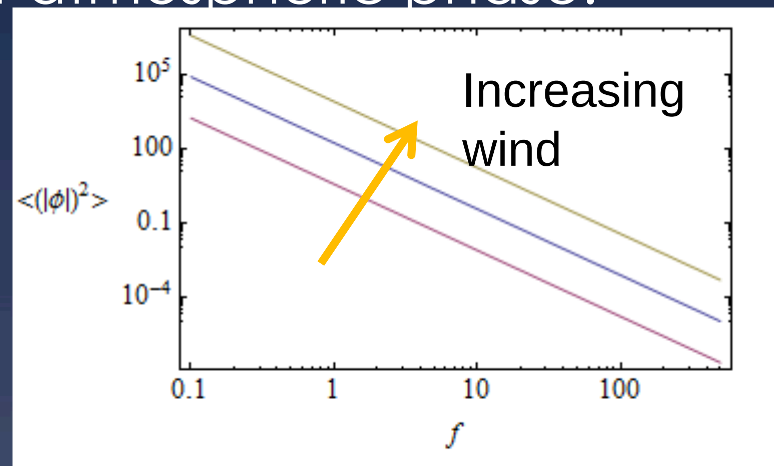
2. Measurement noise in the wavefront sensor

Optimize the controller for best overall performance by varying design parameters such as gain and sample rate

Atmospheric turbulence

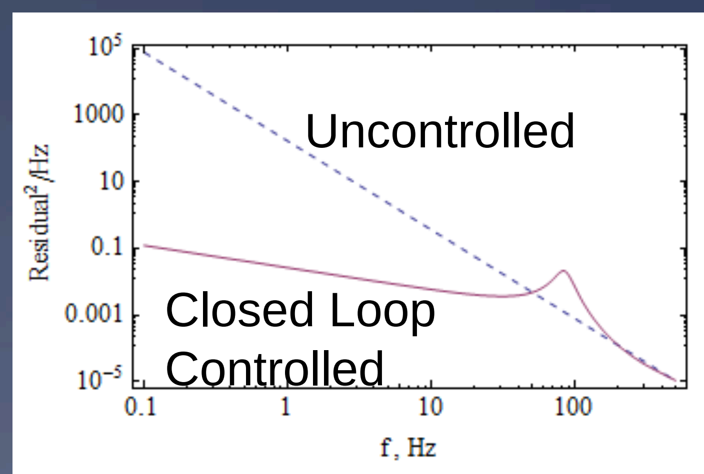
* Temporal power spectrum of atmospheric phase:

$$S_\phi(f) = 0.077\,(v/r_0)^{5/3} f^{-8/3}$$



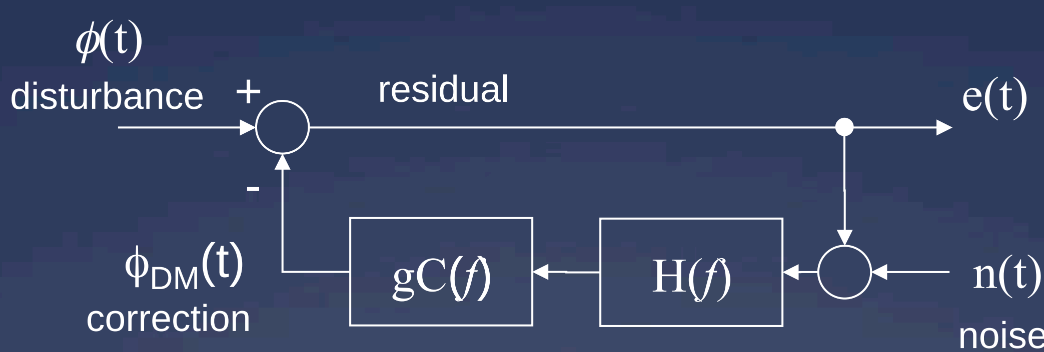
* Power spectrum of residual phase

$$S_e(f) = |\,1/(1 + g\,C(f)\,H(f))\,|^2\, S_\phi(f)$$



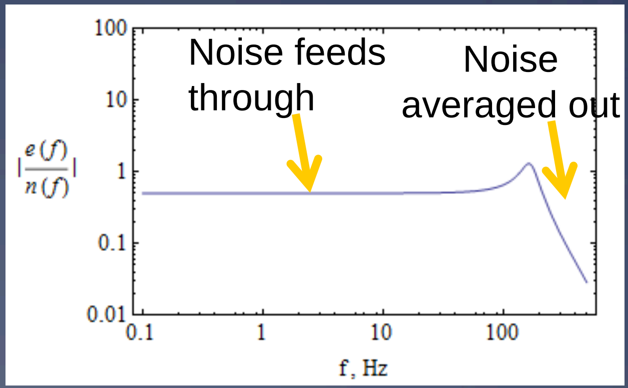
Noise

* Measurement noise enters in at a different point in the loop than atmospheric disturbance



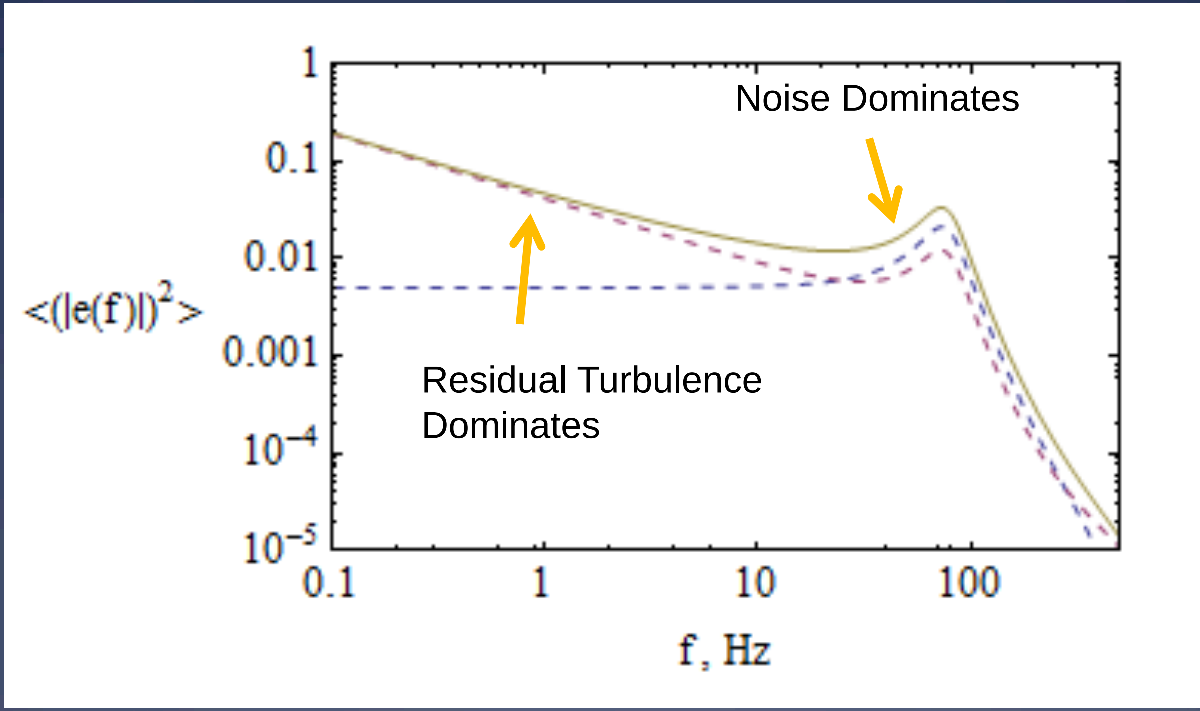
* Closed loop transfer function for noise:

$$e(f) = \frac{gC(f)H(f)}{1 + gC(f)H(f)} n(f)$$



Residual from atmosphere + noise

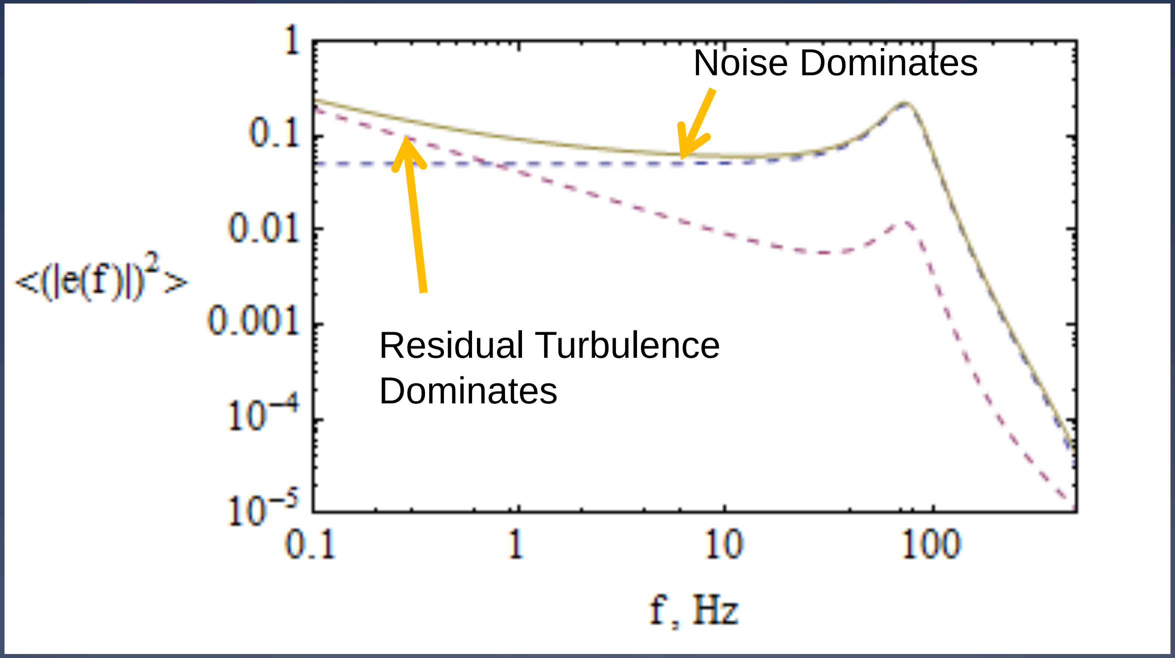
* Conditions
  * RMS uncorrected turbulence: 5400 nm
  * RMS measurement noise: 126 nm
  * gain = 0.4



* Total Closed Loop Residual = 118 nm RMS

Increased Measurement Noise

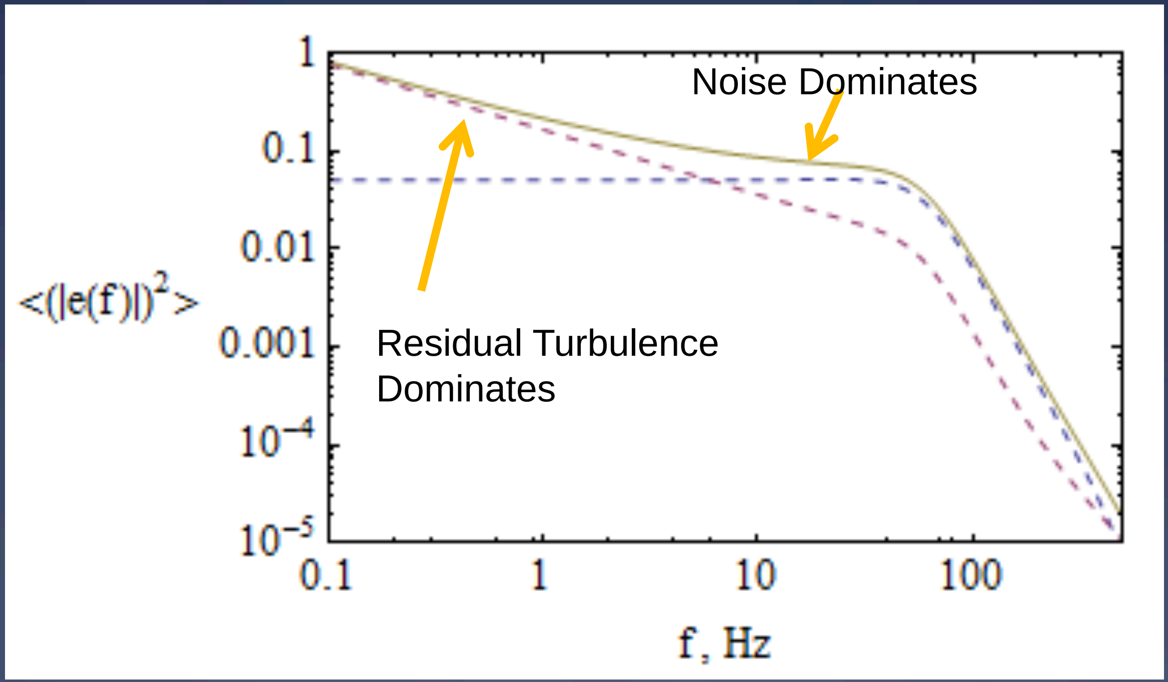
- Conditions
  - RMS uncorrected turbulence: 5400 nm
  - RMS measurement noise: 397 nm
  - gain = 0.4



- Total Closed Loop Residual = 290 nm RMS

Reducing the gain in the higher noise case improves the residual

- Conditions
  - RMS uncorrected turbulence: 5400 nm
  - RMS measurement noise: 397 nm
  - gain = 0.2



- Total Closed Loop Residual = 186 nm RMS

What we have learned

- Pros and cons of feedback control systems
- The use of the Laplace transform to help characterize closed loop behavior
- How to predict the performance of the adaptive optics under various conditions of atmospheric seeing and measurement signal-to-noise
- A bit about loop stability, compensators, and other good stuff

References

We have described feedback control only for AO systems.
For an introduction to control of general systems, some good texts are:

G. C. Goodwin, S. F. Graebe, and M. E. Salgado, “Control System Design”, Prentice Hall, 2001

G. F. Franklin, J. D. Powell, and A. Emami-Naeini, “Feedback Control of Dynamic Systems”, 4th ed., Prentice Hall 2002.

For further information on control systems research in AO, see the CfAO website publications and their references.